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On Penrose limits and gauge theories

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Abstract: We discuss various Penrose limits of conformal and nonconformal backgrounds. In $AdS_5 \times T^{1,1}$, for a particular choice of the angular coordinate in $T^{1,1}$ the resulting Penrose limit coincides with the similar limit for $AdS_5 \times S^5$. In this case an identification of a subset of field theory operators with the string zero mode creation operators is possible. For another limit we obtain a light-cone string action that resembles a particle in a magnetic field. We also consider three different types of backgrounds that are dual to nonconformal field theories: The Schwarzschild black hole in AdS_5, D3-branes on the small resolution of the conifold and the Klebanov-Tseytlin background. We find that in all three cases the introduction of nonconformality renders a theory that is no longer exactly solvable and that the form of the deformation is universal. The corresponding world sheet theory in the light-cone gauge has a $\tau = x^+$ dependent mass term.

Keywords: Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Recently the idea of Penrose that "Any space-time has a plane wave a limit" [1] has undergone a period of a "Renaissance". In Penrose's own words, this limit is the adaptation to pseudo riemannian manifolds of the well-understood procedure of taking tangent space limit. The main difference being that whether T_p in essentially flat, the Penrose limit applied to a null geodesic results in curved space — a plane wave. The idea has been extended, generalized and applied to the study of string theories on the corresponding backgrounds. In particular several papers have been devoted to a detailed analysis of the application of the limit to supergravity backgrounds [2, 3, 4], and to explicitly solving the superstring theory defined on the ppw background [5] as well as understanding the relation between the string spectrum and massless supergravity modes [6]. In [7] the string spectrum of a pp-wave background was shown to arise from the large-N limit (large $g^2_{\rm YM}N$, fixed $g^2_{\rm YM}N/J^2$) of $\mathcal{N} = 4$ SYM theory in 4d. This was accomplished by summing a subset of planar diagrams. The work of Berenstein, Maldacena and Nastase is a very interesting extension of the original AdS/CFT correspondence [8] in that it considers massive string modes.

The goal of this paper is two folded: to apply the Penrose limit to other conformal and non conformal backgrounds and to examine the question of whether the procedure introduced in [7] can be used to identify a string spectrum in certain classes of operators of gauge theories which have $\mathcal{N} = 1$ supersymmetry and are non-conformal theories. We construct various limits of the $AdS_5 \times T^{1,1}$ background. For one particular choice of a geodesic line the resulting Penrose-Güven limit coincides with the similar limit for $AdS_5 \times S^5$. Another limit associated with a different geodesic yields a string theory resembling a particle in a magnetic field. We then analyze the limits of backgrounds that are not dual to conformal field theories: The Schwarzschild black hole in AdS_5, D3-branes on the small resolution of the conifold and the Klebanov-Tseytlin background.

We find that in all these cases there exist a limit which results in a string theory described by a worldsheet action that includes a mass term that depends on the worldsheet time. We make the first steps toward the identification of the field theory operators that correspond to the low lying string states.

While this paper was being prepared for publication two manuscripts [9] and [10] that discuss similar questions appeared on the net. These papers overlap with our discussions of the limits in the $T^{1,1}$ and $T^{p,q}$ models as well as the identification of the string spectrum in the field theory picture.

The paper is organized as follows. In section 2 various Penrose limits of conformal backgrounds are taken. In particular in the context of $AdS_5 \times T^{1,1}$ we identify a limit which reproduce the exact geometry found in [7] for the $AdS_5 \times S^5$ case and describe another limit which leads to a string background resembling a particle in a magnetic field. We then discuss the general $T^{p,q}$ background. Section 2 is devoted to the analysis of several non conformal cases. These include the AdS black hole, the small resolution of the conifold [11] and the Klebanov Tseytlin model [12]. We then comment on the general structure of the Penrose limit of the non-conformal backgrounds. In the appendix we discuss the local nature of the limit we take in the context of conformal backgrounds.

2. Penrose limits in conformal backgrounds

The analysis of [7] is completely symmetric with respect to the choice of the U(1) coordinate inside S^5 or equivalently the U(1) subgroup of SU(4) R-symmetry. In the case of $AdS_5 \times T^{1,1}$ there is a clear difference between the three possible U(1) coordinates; only one correspond to the U(1) R-symmetry. To clarify this difference we next study various possibilities.

2.1 PP wave limit on $AdS_5 \times T^{1,1}$

We consider various Penrose limits in the geometry of $AdS_5 \times T^{1,1}$

$$\begin{aligned} R^{-2} ds^2 \;=\; & -dt^2 \cosh^2\rho + d\rho^2 + \sinh^2\rho d\Omega_3^2 + \frac{1}{9}\left[d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2\right]^2 + \\ & + \frac{1}{6}\left[d\theta_1^2 + \sin^2\theta_1^2\phi_1^2 + d\theta_2^2 + \sin^2\theta_2 d\phi_2^2\right] . \end{aligned} \tag{2.1}$$

In analogy with [7] we concentrate on the motion of a particle that moves along a direction given by $\psi + \phi_1 + \phi_2$ in a geodesic defined in a small neighborhood of $\rho = 0$ and $\theta_i = 0$. Technically this amounts to changing the metric to new coordinates

$$\begin{aligned}
\tilde{x}^+ &= \frac{1}{2}\left[t + \frac{1}{3}(\psi + \phi_1 + \phi_2)\right], \\
\tilde{x}^- &= \frac{1}{2}\left[t - \frac{1}{3}(\psi + \phi_1 + \phi_2)\right], \\
\Phi_1 &= \phi_1 - \frac{1}{2}\left[t + \frac{1}{3}(\psi + \phi_1 + \phi_2)\right], \\
\Phi_2 &= \phi_2 - \frac{1}{2}\left[t + \frac{1}{3}(\psi + \phi_1 + \phi_2)\right],
\end{aligned} \tag{2.2}$$

and subsequently we take the $R \to \infty$ limit with

$$x^+ = \tilde{x}^+, \qquad x^- = R^2 \tilde{x}^-, \qquad \rho = \frac{r}{R}, \qquad \theta_i = \sqrt{6}\,\frac{r_i}{R}. \tag{2.3}$$

In this limit the metric becomes

$$\begin{aligned}
ds^2 &= -4dx^+dx^- - \mu^2(r^2 + r_1^2 + r_2^2)(dx^+)^2 + \\
&\quad + dr^2 + r^2 d\Omega_3^2 + dr_1^2 + r_1^2 d\Phi_1^2 + dr_2^2 + r_2^2 d\Phi_2^2,
\end{aligned} \tag{2.4}$$

where we have introduced the mass parameter μ as a rescaling $x^+ \to \mu x^+$ and $x^- \to x^-/\mu$. Since each pair (r_i, Φ_i) parametrizes an R^2 we end up with a result exactly matching that of [7], that is, a background of the form

$$\begin{aligned}
ds^2 &= -4dx^+dx^- - \mu^2 \vec{z}^{\,2}(dx^+)^2 + d\vec{z}^{\,2}, \\
F_{+1234} &= F_{+5678} \sim \mu.
\end{aligned} \tag{2.5}$$

This background has been studied as an exactly solvable string theory in Ramond-Ramond background [5, 6].

2.2 Other limits of $AdS_5 \times T^{1,1}$, a magnetic case

The combination of variables that we took in the previous subsection was rather particular and was dictated by field theory considerations which we will discuss in more detail in section 4. It is also natural, from the geometrical point of view to consider other limits. In particular it is natural to consider a limit of particles moving along the ψ or ϕ_i directions. We will see that they look rather different. Introducing in (2.1) the following change of variable

$$\tilde{x}^+ = \frac{1}{2}\left(t + \frac{1}{3}\psi\right), \qquad \tilde{x}^- = \frac{1}{2}\left(t - \frac{1}{3}\psi\right) \tag{2.6}$$

and further taking the limit

$$\begin{aligned}
x^+ &= \tilde{x}^+, \qquad & x^- &= R^2\tilde{x}^-, \qquad \rho = \frac{r}{R}, \\
\theta_i &= \frac{\pi}{2} + \sqrt{6}\,\frac{y_i}{R}, \qquad & \phi_i &= \sqrt{6}\,\frac{x_i}{R}.
\end{aligned} \tag{2.7}$$

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we obtain the following metric resembling the motion of a particle in a magnetic field.

$$\begin{aligned} ds^2 &= -4dx^+(dx^- + \mu y_1 dx_1 + \mu y_2 dx_2) - \mu^2 \vec{r}^{\,2}(dx^+)^2 + d\vec{r}^{\,2} + d\vec{y}^2 + d\vec{x}^2 \,, \\ F_5 &= \mathcal{F}_5 + *\mathcal{F}_5 \,, \qquad \mathcal{F}_5 \sim \mu\, dx^+ \wedge dy^1 \wedge dx^1 \wedge dy^2 \wedge dx^2 \,, \end{aligned} \tag{2.8}$$

where $\vec{y} = (y_1, y_2)$ and $\vec{x} = (x_1, x_2)$. This background can be transformed into (2.5) by means of an appropriate coordinate transformation. However, from the field theory point of view (see discussion in subsection 4.4) these two limits seem to be distinct.

The gauge fixed light-cone bosonic string action in this case is

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma \left[\frac{1}{2}\partial_\alpha \vec{r} \partial^\alpha \vec{r} + \frac{1}{2}\partial_\alpha \vec{y} \partial^\alpha \vec{y} + \frac{1}{2}\partial_\alpha \vec{x} \partial^\alpha \vec{x} - \frac{1}{2}\mu^2 \vec{r}^2 + 2\mu \vec{y} \cdot \partial_\tau \vec{x} \right], \tag{2.9}$$

which is exactly solvable.

Finally we consider motion along the ϕ_1, as before we introduce

$$\tilde{x}^+ = \frac{1}{2}\left(t + \frac{1}{\sqrt{6}}\phi_1\right), \qquad \tilde{x}^- = \frac{1}{2}\left(t - \frac{1}{\sqrt{6}}\phi_1\right) \tag{2.10}$$

and further taking the limit

$$\begin{aligned} x^+ &= \tilde{x}^+ \,, \qquad & x^- &= R^2 \tilde{x}^- \,, \qquad & \rho &= \frac{r}{R} \,, \\ \theta_1 &= \frac{\pi}{2} + \sqrt{6}\,\frac{y_1}{R} \,, \qquad & \theta_2 &= \frac{\pi}{2} + \sqrt{6}\,\frac{y_2}{R} \,, \qquad & \phi_2 &= \sqrt{6}\,\frac{x_2}{R} \,, \qquad \psi = 3\frac{z}{R} \,. \end{aligned} \tag{2.11}$$

we obtain the following metric resembling the motion of a particle in a magnetic field.

$$\begin{aligned} ds^2 &= -4dx^+(dx^- + \mu y_1 dz) - \mu^2(\vec{r}^{\,2} + 2y_1^2)(dx^+)^2 + d\vec{r}^{\,2} + d\vec{y}^2 + dx_2^2 + dz^2, \\ F_5 &= \mathcal{F}_5 + *\mathcal{F}_5 \,, \qquad \mathcal{F}_5 \sim \mu\, dx^+ \wedge dz \wedge dy^1 \wedge dy^2 \wedge dx^2 \,, \end{aligned} \tag{2.12}$$

where $\vec{y} = (y_1, y_2)$.

2.3 PP wave limit on $AdS_5 \times T^{p,q}$

In this section we discuss the analogous problem for the motion of a particle in $T^{p,q}$ space. We thus consider a solution to the IIB equations of motion given by $AdS_5 \times T^{p,q}$ where the metric on $T^{p,q}$ is given by

$$\begin{aligned} ds^2_{T^{p,q}} &= \lambda_1^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \lambda_2^2(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) + \\ &+ \lambda^2(d\psi + p\cos\theta_1 d\phi_1 + q\cos\theta_2 d\phi_2)^2 \,. \end{aligned} \tag{2.13}$$

This space is Einstein provided:

$$\frac{1}{\lambda_1^2}\left[1 - \frac{\lambda^2 p^2}{2\lambda_1^2}\right] = \frac{1}{\lambda_2^2}\left[1 - \frac{\lambda^2 q^2}{2\lambda_2^2}\right] = \frac{\lambda^2}{2}\left[\frac{p^2}{\lambda_1^4} + \frac{q^2}{\lambda_2^4}\right] \tag{2.14}$$

In this sense the metric considered in the previous section is a particular case of this more

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general space corresponding to $p = q = 1$, $\lambda_1^2 = \lambda_2^2 = 1/6$ and $\lambda^2 = 1/9$. The element of this class of spaces that have attracted the most attention is precisely $T^{1,1}$ since it is the only that provides a supersymmetric solution of IIB [13].

Next we perform the following coordinate transformation

$$\begin{aligned}
\tilde{x}^+ &= \frac{1}{2}\left[t + \Lambda\lambda(\psi + \phi_1 + \phi_2)\right], \\
\tilde{x}^- &= \frac{1}{2}\left[t - \Lambda\lambda\frac{1}{3}(\psi + \phi_1 + \phi_2)\right], \\
\Phi_1 &= \phi_1 - \frac{\lambda p}{4\Lambda\lambda_1^2}\left[t + \Lambda\lambda(\psi + \phi_1 + \phi_2)\right], \\
\Phi_2 &= \phi_2 - \frac{\lambda q}{4\Lambda\lambda_2^2}\left[t + \Lambda\lambda(\psi + \phi_1 + \phi_2)\right].
\end{aligned} \tag{2.15}$$

Here we have introduced a constant Λ which is used to matched the radii of AdS_5 and $T^{p,q}$. Taking the large R limit in the form

$$x^+ = \tilde{x}^+, \qquad x^- = R^2\tilde{x}^-, \qquad \rho = \frac{r}{R}, \qquad \theta_i = \frac{r_i}{\Lambda\lambda_i R}. \tag{2.16}$$

we obtain

$$\begin{aligned}
ds^2 &= -4dx^+dx^- - \mu^2\left(r^2 + \frac{\lambda^2 p^2}{4\Lambda^2\lambda_1^4}r_1^2 + \frac{\lambda^2 q^2}{4\Lambda^2\lambda_2^4}r_2^2\right)(dx^+)^2 + \\
&\quad + dr^2 + r^2 d\Omega_3^2 + dr_1^2 + r_1^2 d\phi_1^2 + dr_2^2 + r_2^2 d\phi_2^2.
\end{aligned} \tag{2.17}$$

Note that each pair (r_i, ϕ_i) parametrizes an $\mathbf{R}^2$. At the string theory level we can identify, in principle, three different masses for the eight transverse coordinates. Although here we are considering a conformal background, this splitting is characteristic in nonconformal situations, where different coordinates get different masses due to the introduction of nonconformality.

3. Nonconformal cases

One of the most striking features of the AdS/CFT is that it allows to go beyond its original statement about the duality between IIB string theory in $AdS_5 \times S^5$ and $\mathcal{N} = 4$ large N super Yang-Mills. From phenomenological perspective backgrounds that are not conformal invariant play a very important role. In this section we consider the Penrose limit of three different types of nonconformal backgrounds. Namely, we consider the Schwarzschild black hole in AdS_5 complemented by S^5 and a nontrivial 5-form; in this case the scale in the problem is provided by the mass of the Schwarzschild black hole. We also consider the Maldacena limit of D3-branes placed at the origin of the small resolution of the conifold, which is a natural deformation of the $AdS_5 \times T^{1,1}$ solution considered before. Finally in this section we consider the simplest of the theories with varying 5-form flux — the Klebanov-Tseytlin solution [12]. As oppose to the conformal cases discussed before non of this deformation allows for exact solvability at the string theory level due to an explicit dependence on the metric on the x^+ coordinate. However, it is possible to extract some information about the field theory side.

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3.1 Penrose limit of the Schwarzschild black hole in AdS

In this section we consider the Penrose limit for the Schwarzschild black hole in AdS. This deformation of the AdS/CFT, when analytically continued to euclidean time is related to a nonzero temperature deformation in the field theory side. However, here we consider strictly the lorentzian signature since it is a requirement in the implementation of the Penrose limit [1]. The metric we consider in this subsection is[1]

$$R^{-2}ds^2 = -r^2\left(1-\frac{r_0^4}{r^4}\right)dt^2 + \frac{dr^2}{r^2(1-r_0^4/r^4)} + r^2 d\Omega_3^2 + d\psi^2 + \sin^2\psi d\Omega_4^2\,. \tag{3.1}$$

Before applying to Penrose limit we bring the metric into a convenient form following [1], [3]. We study the null geodesic determined by the following effective lagrangian

$$\mathcal{L} = -r^2\left(1-\frac{r_0^4}{r^4}\right)\dot{t}^2 + \dot{\psi}^2 + \frac{\dot{r}^2}{r^2(1-r_0^4/r^4)}\,, \tag{3.2}$$

where dot represents derivative respect to the affine parameter. Since the effective lagrangian does not depend explicitly on the coordinates t and ψ we have two integrals of motion:

$$\dot{\psi} = \mu\,, \qquad \dot{t} = \frac{E}{r^2(1-r_0^4/r^4)}\,. \tag{3.3}$$

For null geodesics $\mathcal{L}=0$ we obtain an equation for r

$$\dot{r}^2 + \mu^2 r^2\left(1-\frac{r_0^4}{r^4}\right) = E^2\,. \tag{3.4}$$

We choose the affine parameter u as part of a new coordinate system (u,v,ϕ) in which we can enforce $g_{uu}=g_{u\phi}=0$ and $g_{uv}=1$. In these coordinates the metric takes the form

$$\begin{aligned} R^{-2}ds^2 &= 2dudv + 2\mu^2 r^2\left(1-\frac{r_0^4}{r^4}\right)dvd\phi - r^2\left(1-\frac{r_0^4}{r^4}\right)dv^2 + \left[1-\left(1-\frac{r_0^4}{r^4}\right)\right]d\phi^2 + \\ &\quad + r^2 d\Omega_3^2 + \sin^2(\phi+\mu u)d\Omega_4^2\,, \end{aligned} \tag{3.5}$$

with

$$\frac{dr}{du} = \sqrt{1-\mu^2 r^2\left(1-\frac{r_0^4}{r^4}\right)}\,. \tag{3.6}$$

We now perform the Penrose limit by sending $R\to\infty$ with

$$u\to u\,, \qquad v\to\frac{v}{R^2}\,, \qquad Y^i\to\frac{y^i}{R}\,, \tag{3.7}$$

where Y^i represent a subset of the rest of the coordinates. We find the Penrose limit for the Schwarzschild black hole in AdS to be

$$ds^2 = 2dudv + (1-\mu^2\rho^2(u))d\phi^2 + r^2(u)ds^2(\mathbf{R}^3) + \sin^2(\mu u)ds^2(\mathbf{R}^4)\,, \tag{3.8}$$

[1]For simplicity we consider a particular limit of the Schwarzschild-AdS in global coordinates. The result, however, does not depend on taking this limit. Moreover the result coincides with the Penrose limit of nonextremal D3-branes.

where

$$\rho^2 = r^2\left(1-\frac{r_0^4}{r^4}\right). \tag{3.9}$$

As noted in [3] for ($r_0 = 0$) the Penrose limit is flat space if $\mu \equiv 0$; for $r_0 = 0$ we recover the result corresponding to the Penrose limit in $AdS_5 \times S^5$. The above result is presented in Rosen coordinates we can transform the metric into the usual Brinkman coordinates in which case we obtain:

$$ds^2 = -4dx^+dx^- - \mu^2\left[\left(1-\frac{3r_0^4}{r^4}\right)\phi^2 + \left(1-\frac{r_0^4}{r^4}\right)z_3^2 + z_4^2\right](dx^+)^2 + d\phi^2 + dz_3^2 + dz_4^2\,, \tag{3.10}$$

where z_3 parametrizes $\mathbf{R}^3$ and z_4 parametrizes $\mathbf{R}^4$. In this coordinate system

$$x^+ = \frac{1}{2\mu}\arctan\left[\frac{2\mu^2r^2-1}{2\mu\sqrt{r^2-\mu^2r^4+\mu^2r_0^4}}\right]. \tag{3.11}$$

Note that the coordinates parametrizing $\mathbf{R}^4$ originate from the S^5 and are not affected by the nonconformality introduced by the Schwarzschild-AdS black hole. The masses of the bosons that have been affected are to those directions lying within AdS.

3.2 The small resolution of the conifold

In this subsection we consider a very different type of nonconformality from the previous section. Namely, we consider the Maldacena limit of regular D3-branes on the small resolution of the conifold. A natural scale is introduced in the problem by the radius (minimal volume) of the nonvanishing S^2. The field theory interpretation of this solution was discussed by Klebanov and Witten in [14], the supergravity construction was presented in [11]. The near horizon limit is of the standard D3-brane form

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}ds_6^2\,, \tag{3.12}$$

where ds_6^2 is the metric of the small resolution of the conifold

$$\begin{aligned} ds_6^2 &= \kappa^{-1}dr^2 + \frac{1}{9}\kappa r^2 e_\psi^2 + \frac{1}{6}r^2\left(e_{\theta_1}^2 + e_{\phi_1}^2\right) + \frac{1}{6}(r^2+6a^2)\left(e_{\theta_2}^2 + e_{\phi_2}^2\right),\\ \kappa &\equiv \frac{r^2+9a^2}{r^2+6a^2}\,, \qquad e_{\theta_i} = d\theta_i\,, \qquad e_{\phi_i} = \sin\theta_i d\phi_i\,. \end{aligned} \tag{3.13}$$

As usual the warp factor is a harmonic function on the transverse 6-d space:

$$h = \frac{R^4}{9a^2r^2}\left[1 - \frac{r^2}{9a^2}\ln\left(1+\frac{9a^2}{r^2}\right)\right]. \tag{3.14}$$

Proceeding as in the previous cases, we study the geodesic line along (r,t,ψ) in order to find a natural transformation into coordinates (u,v,x) convenient to perform the Penrose limit. Namely, we consider the null geodesic described by

$$\mathcal{L} = -\frac{3ar}{\sqrt{1-\frac{r^2}{9a^2}\ln(1+\frac{9a^2}{r^2})}}\dot{t}^2 + \frac{\sqrt{1-\frac{r^2}{9a^2}\ln(1+\frac{9a^2}{r^2})}}{3ar}\kappa^{-1}\dot{r}^2 + \frac{\sqrt{1-\frac{r^2}{9a^2}\ln(1+\frac{9a^2}{r^2})}}{3a}\kappa r\dot{\psi}^2\,. \tag{3.15}$$

The equations following from this lagrangian are:

$$
\begin{aligned}
\dot{t} &= \frac{\sqrt{1-\frac{r^2}{9a^2}\ln(1+\frac{9a^2}{r^2})}}{3ar}\,, \\
\dot{\psi} &= \frac{3a\mu}{\kappa\, r\sqrt{1-\frac{r^2}{9a^2}\ln(1+\frac{9a^2}{r^2})}}\,, \\
\dot{r}^2 &= \frac{r^2+9a^2}{r^2+6a^2} - \frac{9a^2\mu^2}{1-\frac{r^2}{9a^2}\ln(1+\frac{9a^2}{r^2})}\,.
\end{aligned} \tag{3.16}
$$

We take the Penrose-Güven limit as

$$
u = u\,, \qquad v \to \frac{v}{R^2}\,, \qquad x \to \frac{x}{R}\,, \qquad \theta_i = \sqrt{6}\,\frac{r_i}{R}\,, \tag{3.17}
$$

resulting in

$$
\begin{aligned}
ds^2 &= 2dudv + \frac{\kappa\,\Lambda}{3a}\left[1-\frac{9a^2\mu^2}{\kappa\,\Lambda^2}\right]dx^2 + \frac{\Lambda r}{3a}ds^2(\mathbf{R}_1^2) + \frac{\Lambda r}{3a}\left(1+\frac{6a^2}{r^2}\right)ds^2(\mathbf{R}_2^2)\,, \\
\Lambda &= \sqrt{1-\frac{r^2}{9a^2}\ln\left(1+\frac{9a^2}{r^2}\right)}\,,
\end{aligned} \tag{3.18}
$$

where $\mathbf{R}_i^2$ represents the $\mathbf{R}^2$ parametrize by (r_i, ϕ_i). This metric can be brought to Brinkman coordinates following the prescription of [3]. As in the previous case this metric smoothly goes to the maximally supersymmetric pp-wave as the conformality parameter goes to zero ($a \to 0$).

3.3 The Klebanov-Tseytlin solution

The Klebanov-Tseytlin solution describes the geometry of a collection of regular and fractional branes on the conifold [12]. It contains a naked singularity in the IR where it must be replaced by the Klebanov-Strassler solution corresponding to the replacement of the conifold by the deformed conifold [15]. Although not completely accurate, the KT geometry provides a simple description of the supergravity dual of the breaking of conformal invariance in the field theory by the introduction of fractional branes, it is also computationally a lot more manageable than the corresponding background for the deformed conifold metric. We thus proceed, with caution, to study the Penrose limit in the KT solution.

In this section we will use the Poincaré coordinates of AdS that are naturally related to the standard D3-brane solution. This approach in principle obscures the relation of the time coordinate to the global time coordinate that we used in the previous sections and that was used in [7]. However, as was explicitly shown in [3], the end result is the same limit and this completely shows the equivalence of both routes. The metric of the KT solution is

$$
\begin{aligned}
ds^2 &= h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}[dr^2 + r^2 ds^2_{T^{1,1}}], \\
h &= \frac{R^4}{r^4}\left(1+P\ln\left(\frac{r}{r_0}\right)\right).
\end{aligned} \tag{3.19}
$$

For the precise normalizations we refer the reader to [16]. For us it will be important the P is proportional to the number of fractional D3-branes and that it is natural to consider it small with respect to the number of regular D3-branes which is proportional to R^4. After a rescaling of the radial coordinate, and similarly r_0, we bring the metric to the form

$$R^{-2}ds^2 = \frac{r^2\eta_{\mu\nu}dx^\mu dx^\nu}{\sqrt{1+P\ln(r/r_0)}} + r^{-2}\sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\,dr^2 + \sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\,ds^2_{T^{1,1}}\,. \tag{3.20}$$

As in the case of $AdS_5\times T^{1,1}$ we concentrate on motion along a geodesic given by $\psi+\phi_1+\phi_2$ in a small neighborhood of $\theta_i = 0$. The effective lagrangian from which the geodesic equation follows is

$$\mathcal{L} = -\frac{r^2\dot{t}^2}{\sqrt{1+P\ln(\frac{r}{r_0})}} + r^{-2}\sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\,\dot{r}^2 + \sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\,\dot{\psi}^2\,, \tag{3.21}$$

where dot represents derivative with respect to the affine parameter u. Since the lagrangian does not explicitly depend on t and ψ we have two integral of motion:

$$\dot{t} = \frac{E}{r^2}\sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\,, \qquad \dot{\psi} = \frac{\mu}{\sqrt{1+P\ln(r/r_0)}}\,. \tag{3.22}$$

From these two relations we find that

$$\dot{r}^2 + \frac{\mu^2 r^2}{1+P\ln(r/r_0)} = E^2\,. \tag{3.23}$$

Our aim is, following [1, 2, 3], to find new coordinates (u, v, x) satisfying $g_{uu} = 0$, $g_{uv} = 1$ and $g_{ux} = 0$, in which the Penrose-Güven limit is naturally taken. A simple solution satisfying this transformation was given in [3] and can be naturally extended to the case under consideration

$$\partial_u = \dot{r}\partial_r + \dot{t}\partial_t + \dot{\psi}\partial_\psi\,, \qquad \partial_v = -\frac{1}{E}\partial_t, \qquad \partial_x = \mu\partial_t + E\partial_\psi\,. \tag{3.24}$$

From now on we set $E = 1$, as in the previous cases. This system can be integrated. After taking the Penrose limit following

$$u \to u\,, \qquad v \to \frac{v}{R^2}\,, \qquad \theta_i = \sqrt{6}\,\frac{r_i}{R}\,, \qquad x \to \frac{x}{R}\,, \tag{3.25}$$

we obtain the following metric

$$\begin{aligned} ds^2 \;=\; & 2dudv + \frac{r^2}{\sqrt{1+P\ln(r/r_0)}}dx_3^2 + \sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\left[1-\frac{\mu^2r^2}{1+P\ln(r/r_0)}\right]dx^2 + \\ & + \sqrt{1+P\ln\Big(\frac{r}{r_0}\Big)}\,\left[dr_1^2 + r_1^2d\phi_1^2 + dr_2^2 + r_2^2d\phi_2^2\right] - \frac{\mu^2}{\sqrt{1+P\ln(r/r_0)}}(r_1^2+r_2^2)du^2\,, \end{aligned} \tag{3.26}$$

where r and u are related according to

$$u = \int \frac{dr}{\sqrt{1-\frac{\mu^2r^2}{(1+P\ln(r/r_0))}}}\,. \tag{3.27}$$

Note that in the particular case of $P = 0$ we find that the above expression can be explicitly integrated ($r = \mu^{-1}\sin\mu u$) giving the pp-wave in Rosen coordinates [3]. One of the most interesting characteristics of the KT-type backgrounds is the dependence on the 5-form on the radius, associated with the RG-cascade. In the present context this feature remains

$$F_5 = \mathcal{F}_5 + *\mathcal{F}_5, \qquad \mathcal{F}_5 \approx \left(1 + P\ln\Big(\frac{r}{r_0}\Big)\right)\dot{\psi} du\wedge dr_1\wedge r_1 d\phi_1\wedge dr_2\wedge r_2 d\phi_2\,. \tag{3.28}$$

Similarly some of the components of the 3-form fields survive the Penrose-Güven limit

$$\begin{aligned} B_2 &\sim P\ln\Big(\frac{r}{r_0}\Big)(dr_1\wedge r_1 d\phi_1 - dr_2\wedge r_2 d\phi_2)\,, \\ F_3 &\sim P\dot{\psi} du\wedge(dr_1\wedge r_1 d\phi_1 - dr_2\wedge r_2 d\phi_2) \end{aligned} \tag{3.29}$$

3.4 Comments on the general structure of the Penrose limit in nonconformal backgrounds

We have seen that in all the three examples that we discussed the nonconformality parameters appear naturally as a perturbation of the metric away from the exactly solvable pp-wave limit discussed in [5, 6]

$$ds^2 = -4dx^+dx^- + d\vec{z}^2 - \mu^2\left[\sum_i \left(1 + \epsilon f_i(x^+)\right) z_i^2\right](dx^+)^2\,, \tag{3.30}$$

where ϵ is the nonconformality parameter. The collection of functions $f_i(x^+)$ characterize the form of the "mass" deformation for the coordinate z_i; in the case of the Schwarzschild black hole we note that $f_i = 0$ for the directions within the S^5, as expected. From the string theory point of view we have that since the equation of motion for x^- implies that $\Box x^+ = 0$ we can fix the world sheet diffeomorphism invariance by choosing the light cone $x^+ = p^+\tau$. The resulting gauge fixed string theory action will then be interpreted as a theory of eight massive fields with time-dependent mass.

An interesting observation is that the fields z_i can not appear to order higher than two, that is to say, the action is always quadratic in the field z_i. This can be seen by recalling that in the Penrose-Güven limit one rescales $z_i \to z_i/R$ and then multiplies the metric by R^2.

The near horizon limit of nonconformal Dp-branes has been discussed in [17], their Penrose-Güven limit has been presented in [3]. It is interesting to note that they also fall into the general form discussed here, in the sense that they have and x^+-dependent mass function.

4. $\mathcal{N} = 1$ Super Conformal Field Theory interpretation

In this section we attempt to find a field theory interpretation to the limits taken in section 1 for the conifold geometry of $AdS_5 \times T^{1,1}$. We follow closely the procedure used in [7], in the Penrose limit of section 2.1, namely, we first identify the field theory operators that correspond to the ground state with large p^+ and to the first excited level. We then make some preliminary observations about the possibility to have a perturbative expansion that will reproduce the string hamiltonian. We end by discussing the operator identifications of the magnetic cases of section 2.2. It is crucial for us to relate isometries of the $T^{1,1}$ space with charges of the operators of the CFT. To make that connection completely transparent we start with reviewing the construction of the conifold metric following Candelas and De la Ossa [18] and some facts about the superconformal field theory discussed by Klebanov and Witten [19] (we will also rely on the analysis of [21]).

4.1 Review of the the conifold and the superconfomal theory of D3-branes at the conifold singularity

The conifold is defined by the following quadric in $\mathbf{C}^4$:

$$\sum_{i=1}^{4} w_i^2 = 0\,. \tag{4.1}$$

This equation can be written as

$$\det \mathcal{W} = 0\,, \qquad \text{i.e.} \qquad Z_1 Z_2 - Z_3 Z_4 = 0\,, \tag{4.2}$$

$$\mathcal{W} = \frac{1}{\sqrt{2}} \begin{pmatrix} w_3 + i w_4 & w_1 - i w_2 \\ w_1 + i w_2 & -w_3 + i w_4 \end{pmatrix} \equiv \begin{pmatrix} Z_1 & Z_3 \\ Z_4 & Z_2 \end{pmatrix}. \tag{4.3}$$

Equation (4.1) has an $SO(4)$ symmetry that is usually treated as an $\mathrm{SU}(2)\times\mathrm{SU}(2)$. There is also a $\mathrm{U}(1)$ symmetry given by

$$w_i \to e^{i\alpha} w_i\,. \tag{4.4}$$

This last symmetry was identified with the $\mathrm{U}(1)_R$ in the gauge theory side based on the fact that the holomorphic 3-form can be written as

$$\Omega = \frac{dw_1 \wedge dw_2 \wedge dw_3}{w_4}\,, \tag{4.5}$$

and therefore has charge two under this $\mathrm{U}(1)$ symmetry. Moreover, since the chiral superspace coordinate transform as $\Omega^{1/2}$ we obtain that it naturally has charge one.

To write an explicit metric on the conifold we need to find a general solution to eq. (4.3) and assume that the Kähler potential depends only on the radial coordinate which is defined as: $r^2 = \mathrm{tr}(\mathcal{W}^\dagger \mathcal{W})$. The most general solution can be found by acting on a particular solution with elements of $\mathrm{SU}(2)\times\mathrm{SU}(2)$. Namely, given a particular solution Z_0 we construct $\mathcal{W}$ as

$$\frac{\mathcal{W}}{r} = L Z_0 R^\dagger = \begin{pmatrix} a & -\bar{b} \\ b & \bar{a} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{k} & \bar{l} \\ -l & k \end{pmatrix} \tag{4.6}$$

where $|a|^2+|b|^2 = |a|^2+|b|^2 = 1$ and they can be parametrize as $a = \cos\frac{\theta_1}{2}\exp\frac{i}{2}(\psi_1+\phi_1)$, $b = \sin\frac{\theta_1}{2}\exp\frac{i}{2}(\psi_1-\phi_1)$ and similarly for k and l. With this choice of parametrization of $\mathrm{SU}(2)$ we find

$$Z_1 = -r\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\exp\frac{i}{2}(\psi+\phi_1-\phi_2)\,,$$

$$
\begin{aligned}
Z_2 &= r \sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\exp\frac{i}{2}(\psi - \phi_1 + \phi_2)\,,\\
Z_3 &= r \cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\exp\frac{i}{2}(\psi + \phi_1 + \phi_2)\,,\\
Z_4 &= -r \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\exp\frac{i}{2}(\psi - \phi_1 - \phi_2)\,,
\end{aligned}
\tag{4.7}
$$

where $\psi = \psi_1 + \psi_2$. Looking at equation (4.3) we see that the $Z's$ are linear combinations of the $w's$ and thus in order for the latter to have R-charge one we must identify that symmetry with shifts of $\psi = 2\beta$, precisely as in [20].

4.2 Identification of lowest "string modes" of the KW model

The lowest components of the chiral superfields of the KW model are related to the conifold parameters in the following way [19]

$$
Z_1 = A_+B_+\,, \qquad Z_2 = A_-B_-\,, \qquad Z_3 = A_+B_-\,, \qquad Z_4 = A_-B_+\,. \tag{4.8}
$$

where (A_-, A_+) and (B_-, B_+) are doublets of $\mathrm{SU}(2)_A$ and $\mathrm{SU}(2)_B$ global symmetries respectively and carry a $\mathrm{U}(1)_R$ charge of 1/2. The question now is how to characterize these fields in the sector that corresponds to the Penrose limit. In [7] the light-cone hamiltonian was taken to be $2P^- = \Delta - J$ where J is the generator of rotations in the ψ direction, $J = -i\partial_\psi$. Following (2.1) and the discussion above about the identification of $\psi = 2\beta$, it is natural to identify J for the $T^{1,1}$ case as follows

$$
J = -i[1/2\partial_\beta + \partial_{\phi_1} + \partial_{\phi_2}]\,. \tag{4.9}
$$

In table 1 we classify the fields of the KW theory by respect to their gauge transformations, their $\mathrm{U}(1)_A \times \mathrm{U}(1)_B \times \mathrm{U}(1)_R$ charges, where $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_B$ associate with the T_3 generators of $\mathrm{SU}(2)_A$ and $\mathrm{SU}(2)_B$ respectively, their $J = 1/2\,\mathrm{U}(1)_R + \mathrm{U}(1)_A + \mathrm{U}(1)_B$ charges and their conformal dimension. In addition table 1 contain composite operators that carry $\Delta - J = 0, 1$.

From table 1 it is clear that in a similar manner to the $\mathcal{N} = 4$ case [7], the natural candidate in the KW model for the operator that corresponds to the light-cone ground state is

$$
\frac{1}{\sqrt{J}N^{J/2}}\,\mathrm{Tr}[(A_+B_+)^J]\,. \tag{4.10}
$$

The bosonic operators associated with $\Delta - J = 1$ are the D_iZ and ϕ_1 and ϕ_2 defined in table 1. The missing 2 bosonic operators may be associated with non chiral operators composed from complex conjugates of the basic bosonic fields as was pointed out in [22].[2] As for the fermionic operators at "level"one we have the operators defined in table 1 as ψ_1 and ψ_2 and in addition there are the gauginos and the missing operators may be again associated with certain non chiral operators.

[2] We thank O. Aharony for pointing this out to us.

	$\mathrm{SU}_L(N)$	$\mathrm{SU}_R(N)$	$\mathrm{U}_A(1)$	$\mathrm{U}_B(1)$	$\mathrm{U}_R(1)$	J	Δ	$\Delta - J$
(A_+, ψ_+^A)	N	$\bar{N}$	$1/2$	0	$(1/2, -1/2)$	$(3/4, 1/4)$	$(3/4, 5/4)$	$(0, 1)$
$(A_- \psi_-^A)$	N	$\bar{N}$	$-1/2$	0	$(1/2, -1/2)$	$(-1/4, -3/4)$	$(3/4, 5/4)$	$(1, 2)$
$(B_+ \psi_+^B)$	$\bar{N}$	N	0	$1/2$	$(1/2, -1/2)$	$(3/4, 1/4)$	$(3/4, 5/4)$	$(0, 1)$
$(B_- \psi_-^B)$	$\bar{N}$	N	0	$-1/2$	$(1/2, -1/2)$	$(-1/4 - 3/4)$	$(3/4, 5/4)$	$(1, 2)$
λ_L	adj	1	0	0	1	$1/2$	$3/2$	1
λ_R	1	adj	0	0	1	$1/2$	$3/2$	1
$Z \equiv A_+ B_+$	$adj \oplus 1$	$adj \oplus 1$	$1/2$	$1/2$	1	$3/2$	$3/2$	0
$\phi_1 \equiv A_+ B_-$	$adj \oplus 1$	$adj \oplus 1$	$1/2$	$-1/2$	1	$1/2$	$3/2$	1
$\phi_2 \equiv A_- B_+$	$adj \oplus 1$	$adj \oplus 1$	$-1/2$	$+1/2$	1	$1/2$	$3/2$	1
$\psi_1 \equiv A_+ \psi_+^B$	$adj \oplus 1$	$adj \oplus 1$	$1/2$	$-1/2$	0	1	2	1
$\psi_2 \equiv B_+ \psi_+^A$	$adj \oplus 1$	$adj \oplus 1$	$-1/2$	$+1/2$	0	1	2	1

Table 1: $\mathrm{SU}(N)_L \times \mathrm{SU}(N)_R$ transformation properties, global charges and dimensions of chiral fields and gauginos.

4.3 On the "strings" of the KW model

Since the Penrose limit of the $AdS_5 \times T^{1,1}$ is identical to that of the $AdS_5 \times S^5$, it is clear that the bosonic part of the light-cone string action associated with the former limit takes the form [7]

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma \left[\frac{1}{2}\partial_\alpha \vec{z} \partial^\alpha \vec{z} - \mu^2 \vec{z}^2\right]. \tag{4.11}$$

The question now is whether one can show that the KW field theory in the Penrose limit admits a string bit picture which is governed by a discretized hamiltonian that flows in the continuum limit to the hamiltonian associated with (4.11). The Penrose limit now associates with

$$g_s^2 N \to \infty \qquad \frac{g_s^2 N}{J^2} \text{ fixed} \qquad \Delta - J \text{ fixed}\,, \tag{4.12}$$

Note that it is g_s and not g^2_{YM} which is involved in the limit since the later becomes large at the IR fixed point. g_s maps in the field theory language, in a manner that is not completely understood to λ the coupling of the superpotential which is given by

$$W = \frac{\lambda}{2}\epsilon^{ij}\epsilon^{kl}\,\mathrm{Tr}[A_i B_k A_j B_l] \tag{4.13}$$

with $i = +, -$. The corresponding scalar potential takes the form

$$V = G^{r\bar{s}}\partial_r W \partial_{\bar{s}}\bar{W} = \mathrm{Tr}[G^{A_+^\dagger A_+}(B_+ A_- B_-)(B_+^\dagger A_-^\dagger B_-^\dagger)] + \cdots \tag{4.14}$$

where the $\cdots$ corresponds the other terms in $G^{r\bar{s}}$. The Kähler metric [19] which takes the form $\sum_i (\bar{w}_i w_i)^{-1/3} dw_i d\bar{w}_j$ can be rewritten in terms of the A_i and B_i fields, inverted and inserted in the expression for the potential. The potential then will have terms like $(A_+^\dagger A_+)^{2/3}(B_+^\dagger B_+)^{-1/3}|A_- B_- A_+|^2$ of total dimension four.

A key issue is obviously whether one can find a perturbative expansion of this interaction potential. It is hard to believe that such a perturbation is possible especially since the theory is at strong gauge coupling. On the other hand as stayed above the final answer should converge to a free light-cone "massive" string. The resolution of the puzzle how such a complicated potential can lead to the same continuum result as for the $AdS_5 \times S^5$ may involve some non-trivial map that will transform the interaction potential into $\mathrm{Tr}[\hat{Z}\hat{\phi}\hat{\bar{Z}}\hat{\bar{\phi}}]$ where $\hat{Z}$ and $\hat{\phi}$ are dimension one operators of $\Delta - J$ equal to 0 and 1 respectively, like for instance $\hat{Z} = Z^{2/3}$, $\hat{\phi} = \phi/Z^{1/3}$. This type of construction is under current investigation.

4.4 String modes in the magnetic limit

In spite of the fact that the Penrose limits of (2.1) and (2.2) are related by coordinate transformation we will argue that the corresponding light-cone hamiltonians are different and therefore determine different projections in the space of operators. In analogy to using $H_{lc} = 2P^- = \Delta - [(1/2)\,\mathrm{U}(1)_R + \mathrm{U}(1)_A + \mathrm{U}(1)_B]$ for the Penrose limit of section we identify $J = \alpha\,\mathrm{U}_R(1)$, where α is some numerical constant for the limit where $\tilde{x}^+ = \frac{1}{2}(t + \frac{1}{3}\psi)$. It is thus clear that all the A_i and B_i will have the same eigenvalue of $2P^-$. In particular for $\alpha = 3/2$ we have that $\Delta - J = 0$ for all the scalar fields and the gauginos. In this case the vacuum state will be highly degenerate since there will be many operators that are analogous to $\mathrm{Tr}[Z^J]$. This high degeneracy may be related to the degeneracy of the Landau levels. The fermions $\psi^A_\pm$ and $\psi^B_\pm$ have $\Delta - J = 2$

For the case that ϕ_i is replacing ψ in the definition of x^+ then naturally $J = \alpha\,\mathrm{U}(1)_A$. In this case there is less degeneracy since J is distinguish between A_+ and A_- but since the B_i carry zero J charge there is still some degeneracy. For instance for $\alpha = 3$ both $\mathrm{Tr}[A_+B_+]$ and $\mathrm{Tr}[A_+B_-]$ correspond to the string ground state.

A similar situation is also encountered in $AdS_5 \times S^5$. Using different linear combinations of the U(1)'s, implies using different J's and therefore different light-cone hamiltonians H_{lc}. In particular, it is possible to get the magnetic case by taking the sum of the three U(1)'s inside SU(4).

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A. Local versus global existence of geodesic congruence

In this appendix we explain the intrinsically local character of some of the limits taken in the main body of the test. Consider the construction of null geodesic in the plane given by (t, ρ, ψ) in any of the geometries discussed previously of for that matter even in $AdS_5 \times S^5$ we concentrate on the part of the metric having the following form:

$$ds^2 = -dt^2 \cosh^2 \rho + d\rho^2 + d\psi^2 \, . \tag{A.1}$$

The equation of the geodesic in this coordinates follows from the lagrangian

$$\mathcal{L} = -\dot{t}^2 \cosh^2 \rho^2 + \dot{\rho}^2 + \dot{\psi}^2 \,, \tag{A.2}$$

where the dot means derivatives with respect to the affine parameter u. Since there is not explicit dependence on ψ or t we have two integral of motion:

$$\dot{\psi} = \mu \,, \qquad \dot{t} = \frac{E}{\cosh^2 \rho} \,. \tag{A.3}$$

substituting in the condition of null geodesic $\mathcal{L} = 0$ we get

$$\dot{\rho}^2 + \mu^2 = \frac{E^2}{\cosh^2 \rho} \,. \tag{A.4}$$

This equation shows the local character of such geodesic. Namely, in a neighborhood of $\rho = 0$ there is always a solution to this equation with nonzero μ and therefore including ψ. However if we allow ρ to be very large we find that the above equation can always be falsified unless we take $\mu \equiv 0$ in which case the solution is

$$\mu \equiv 0 \,, \qquad \sinh \rho = Eu \,, \qquad t = \arctan Eu - \frac{v}{E} \,. \tag{A.5}$$

which shows that away from a neighborhood of $\rho = 0$ the geodesic line is completely independent of ψ and lies wholly within AdS_5. Further, is was shown in [3], explicitly and using the hereditary properties of Penrose limits, that the Penrose limit on AdS always results in flat space. The key point exploited in the body of the paper is that we can set the size of the small neighborhood of $\rho = 0$ by rescaling by R and up to fourth order in $1/R$ there is a null geodesic nontrivially including a dependence on ψ, which is precisely the one used in the body of the paper.

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